Dynamic Response of a Four-Cylinder Compressor Foundation Considering the Effect of Soil-Foundation Interaction—A Case Study

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ABSTRACT

The dynamic response of machine foundation system depends on several factors such as (1) the soil dynamic properties, (2) the geometric properties of the foundation, (3) the amplitude of the applied dynamic loads, and (4) the frequency of the exciting dynamic force. The main goal of machine foundation design is to keep the foundation response within a specific limit of response in order to enable a satisfactory operation of the machine. If the foundation response exceeds this limit, the foundation will adversely affect the performance of the machine and may damage the machine internals, or cause it not to function properly. Furthermore, the excessive vibrations will impose additional stresses on the machine resulting in an increased unbalance loading and thus leading to increased dynamic loads on the soil-foundation system. This paper presents the results of the dynamic analysis of a four-cylinder compressor foundation. The original design of the foundation was performed in the early 1960s and ignored the effect of the soil in the response of the foundation system, thus, the foundation has been suffering from excessive vibration. The foundation block supports a four-cylinder dress-rand compressor, suction and discharge bottles, a crank and driving motor with a total weight of approximately 219 kips. The results of a three-dimensional finite element model of the soil–foundation system were used to determine the dynamic response of the soil-foundation system and to assess the foundation response under the applied dynamic loading imposed by the compressor crank. The dynamic analysis is performed by: (1) performing eigenvalue analysis of the foundation block, considering the effect of the soil-foundation interaction to determine the soil-foundation natural frequencies and modal participation factors, and (2) performing forced response of the foundation under applied crankshaft unbalance load to determine the forced response amplitude of the soil-foundation system.

KEYWORDS Compressor foundation, soil-structure interaction, finite element method, dynamic loading, machine foundation, stiffness, damping.

INTRODUCTION

Large reciprocating compressors are utilized in various industries including gas, oil, and petrochemical production. The foundation supporting the compressor equipment are subject to high vibrations instigated by the unbalanced machine forces as well as the machine operating speed. Therefore, for the compressor to have a satisfactory operation, the vibrations (dynamic amplitude) of the machine resulting from these dynamic forces should be limited to very small values at the location of the machine anchorage to the foundation. Usually these limits should not exceed a few microns (10 to 12 microns). If the dynamic amplitude at the machine bearings exceeds such limits, excessive vibrations occurs and will damage the machine or cause it not to function properly. Further, these vibrations may adversely affect the building or persons working near the machines unless the frequency and amplitude of the vibrations is controlled. Thus, in order to limit the vibrational amplitudes generated by these machines, the reciprocating compressor foundations are normally built as massive solid concrete blocks doweled to a single mat or to a continuous mat supporting several machines. Depending on the soil properties, the foundation mat can either by supported directly on the soil continuum or supported on piles. The design of these foundations was often conducted using the rule of thumb by increasing the weight of the foundation and/or strengthening the soil beneath the foundation base, which would provide high-tuned supports for the machine. The total weight of foundation for this type of foundation design is usually two to three times greater than the weight of the machine it supports. It was not until the 1950s when the vibration analysis of machine foundations was implemented using the lumped mass approach and based on a theory of a surface load on an elastic half-space. The theory of elastic half-space assumes that the foundation is (1) On the surface of a homogeneous stratum overlying the bedrock, (2) Partially embedded foundations in a homogeneous stratum overlying the bedrock. The elastic half-space ignored the shape of the foundation and assumed that the foundation has a circular contact base. The effect of the foundation geometry was later considered by Kobori (1962) where he determined the dynamic amplitude response in the vertical, lateral, and rocking modes of vibration for a rectangular foundation. Chae (1969) suggested the use of equivalent radius to estimate the response of rectangular foundation.

The quality of the design and construction of the compressor mounting system and the integrity of the foundation for the reciprocating compressor affects its long-term operations reliability. Degradation of the soil-foundation system will result in additional differential displacement in the foundation resulting in misalignment of the compressor shaft and an increase in the unbalance loading on the foundation. This would increase the foundation vibration and eventually would lead to crankshaft failure. A good design for the compressor foundation should not only accommodate all applied loads including the horizontal gas and inertia loads, compressor and foundation weights, thermal loading and the compressor frame distortion, but also, it should reduce the effect of the vibration on the foundation and the soil as well. Due to complexity of the analysis to mitigate the effect of negative vibrations on the compressor foundation, and the interaction of the foundation with the soil response, the finite element tool is usually utilized to perform such analysis and design of the foundation. Since the finite element analysis is a numerical representation of a physical engineering system, therefore, the finite element model should accurately capture the geometric detail of the system, the actual boundary conditions, and the excitation environment of the dynamic system in order to simulate the real behavior of the problem. Generally, a dynamic finite element analysis consists of three major steps: (1) Idealization of geometry, materials, loading and boundary conditions (2) Formulation of stiffness, mass and damping matrices, and (3) Solution of the resulting equations of motion. A fundamental kinematic assumption of all finite element methods is that the displacement field $u(x, y)$ is completely defined by the displacement vector $\{u\}$ of the nodal points of the system. There are several parameters that affect the finite element results, such as element type, element size, boundary conditions and the effect of the soil-structure interaction on the response of the machine-foundation system. During machine oscillation, the machine-foundation system interacts with the soil in two mechanisms that occur simultaneously in minor time lag:

1. Kinematic interaction, which is the difference in motion of the foundation system and the free field motion due to the presence of stiff foundation system, waves inclination, waves incoherence, and foundation embedment.

2. Inertial interaction, which is the additional inertial dynamic forces and displacements that are imposed on the soil-foundation system during machine foundation oscillation.

Both types of interaction must be considered in the machine-foundation system in order to achieve proper design. While some researchers think that ignoring the effect of Soil-Structure Interaction (SSI) is conservative, Kavvadas and Gazetas (1993) suggested that the effect of soil-structure interaction increase structural demands and the forces that result from SSI govern the structural response. These forces should be determined with accurate analyses.

This paper investigates the dynamic response of a four-cylinder Dress-Rand compressor foundation considering the effect of the soil-structure interaction on the foundation dynamic response. The dynamic analysis of the foundation is performed to evaluate the foundation response under applied dynamic loading resulting from the compressor crank. The dynamic analysis is performed by (1) Performing eigenvalue analysis of the foundation block considering the effect of the soil-foundation interaction to determine the soil-foundation natural frequencies and modal participation factors, (2) Performing forced response of the foundation under applied crank unbalance load to determine the forced response amplitude of the soil-foundation system.

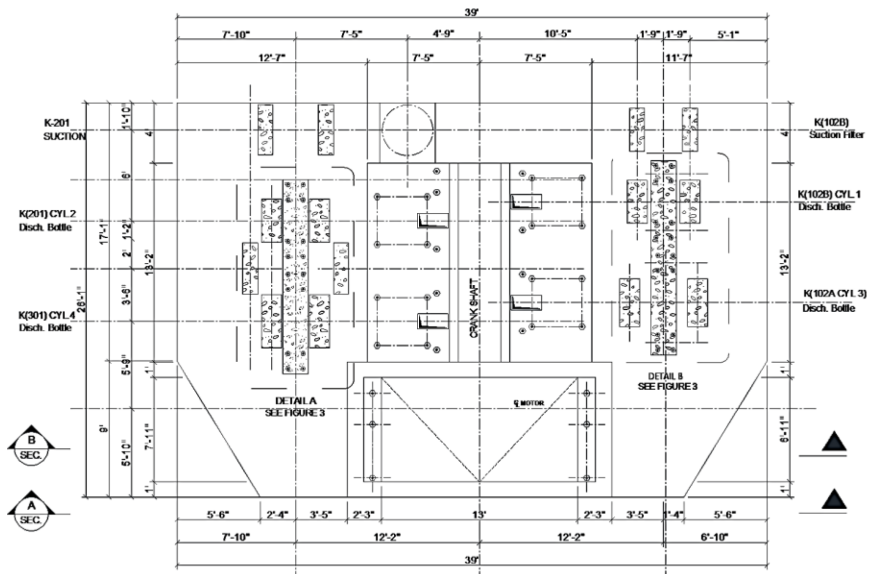

Figure 1. Compressor Foundation Plan View

FOUNDATION GEOMETRY

The foundation block supports a four-cylinder Dress-Rand compressor, suction and discharge bottles, a crank and driving motor with total weights of approximately 219 kips. Figures 1 and 2 show a plan and one section view of the foundation block.

DESIGN INPUT

Soil Data

The main design input parameters affecting the computation of the soil dynamic springs are typically provided by the geotechnical engineer’s report and are as follows:

Soil Shear Wave Velocity

$$V_s = 201 \frac{m}{\sec}$$

Soil Compression Wave Velocity

$$V_p = 1440 \frac{m}{\sec}$$

Soil Mass Density

$$\rho = 2.00 \frac{gm}{cm^3}$$

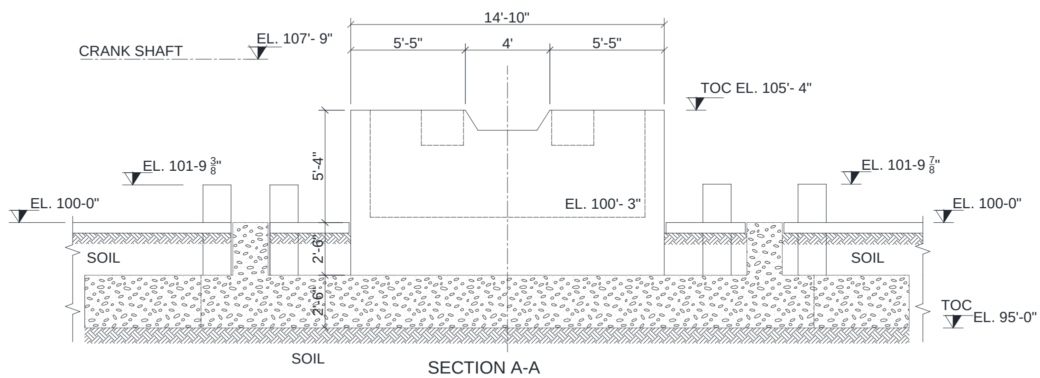

Figure 2. Compressor Foundation Section View

Based on the soil shear and compression wave velocities, the soil dynamic properties are calculated as follows:

Soil Poisson's Ratio

$$\mu = \left[V_p^3 + 2 \cdot V_s^2\right] / \left[2 \cdot \left(V_p^2 - V_s^2\right)\right] \tag{1}$$

Soil Dynamic Shear Modulus

$$G_{dynamic} = \rho \cdot V_s \tag{2}$$

Soil Dynamic Young's Modulus

$$E_{soil} = 2 \cdot \rho \cdot V_s^2 (1 + \mu) \tag{3}$$

The effect of the soil layering on the foundation is a complex phenomenon; additionally, the soil stiffness and damping are highly dependent upon the soil shear modulus as well as the forcing frequency of excitation. To determine the soil stiffness and damping coefficients, the weighted average shear modulus is evaluated based on the elastic half-space theory and is presented in equation (4) as follows:

$$G_{soil} = \left[\sum_{i=1}^{n} \frac{h_i}{A_i}\right] \Bigg/ \left[\sum_{i=1}^{n} \frac{h_i}{A_i \cdot G_i}\right] \tag{4}$$

Where, h_i is thickness of the soil i^{th} soil layer, G_i is the shear modulus of the soil i^{th} soil layer, A_i is the area of stress influence of a horizontal plane (spreading below the foundation at a ratio of 2:1) measured at the center of i^{th} soil layer and n is the number of soil layers to a depth equal to one diameter or one long dimension of foundation, whichever is greater

Machine Data

The weight of the compressor component assembly used in the analysis are as shown in Table 1.

Table 1: Weight of Compressor Components Assembly

Weight of the Motor Rotor:	$Wt_{mot} = 17.8kip$
Weight of the Running Gear:	$Wt_{gear} = 150kip$
Weight of the Cylinders:	$Wt_{cyl} = 18kip$
Weight of the Suction Bottles:	$Wt_{scu} = 6.2kip$
Weight of the Recycle Bottle:	$Wt_{rec} = 7.5kip$
Weight of the Motor Stator and other Density Parts:	$Wt_{mot} = 20.6kip$
Weight of the Discharge Bottles:	$Wt_{dis} = 9.52kip$

Concrete Properties

The member's material properties used in the finite element model are based on the steel properties for the steel framing and the concrete compressive strength. The material properties for the foundation base mat shell elements and the foundation piers are calculated as follows:

Concrete Young's Modulus

$$E_c = 4700\sqrt{f_c}\ \text{(MPa)} \tag{5}$$

Concrete Shear Modulus

$$G_s = \frac{E_c}{2\cdot(1+\mu)} \tag{6}$$

Soil Mass Density

$$\rho = 2.00\frac{gm}{cm^3}$$

Where f_c = concrete compressive strength = 20.684 MPa (3000psi)

Where μ = Poisson's ratio for concrete = 0.17

MODELING OF THE SOIL-FOUNDATION SYSTEM

To determine the dynamic properties of the soil-foundation system, a three-dimensional finite element model was created using the finite element code of ANSYS 13 (2011). The soil continuum was modeled using the ANSYS three-dimensional brick element "SOLID186". ANSYS element "SOLID186" is a 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes that have three degrees of freedom per node, which are translations in the nodal x, y, and z directions.

The running time and accuracy of a finite element solution is greatly affected by the mesh quality. Several recommendations are provided in the literature in order to set the element size for wave propagation analyses using the finite element method. The general concept is that the mesh should be fine enough to resolve the propagating wave. The size of the element, S_e, was chosen based on the recommendation of Lysmer et al. (1969) that the size of elements must be based on the maximum frequency content of the applied loads. Lysmer et al. (1969) proposed the following criteria for selecting the finite element size:

$$S_e \approx 0.2\lambda_{shear} \tag{7}$$

Where

S_e = finite element size, and λ_{shear} = soil shear wavelength (m).

Figure 3 shows finite element model (discretized model) of the compressor foundation.

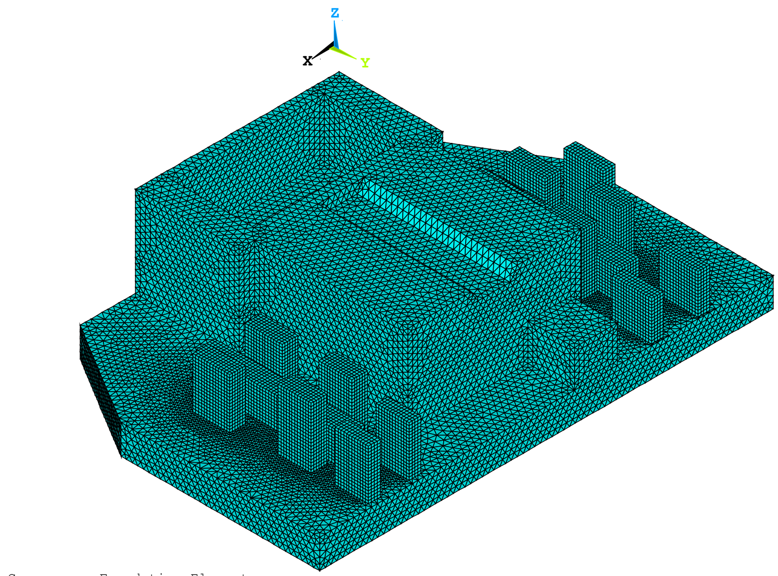

Figure 3. Compressor Foundation Finite Element Model

The flexibility of the soil supporting media is incorporated in the analysis by considering translational springs in the three orthogonal directions attached at the base mat finite element nodes. The effect of the soil on the response of the compressor foundation is captured by modeling the soil dynamic elastic properties using spring-damper elements. ANSYS element COMBIN14 was used to the model the soil vertical and lateral stiffness.

The element is defined by two nodes, a spring constant (K) and damping coefficients (Cv)

The element longitudinal spring constant has a unit of (Force/Length); the damping coefficient units are (Force Time/Length). The element behavior is controlled using the element Key Options; the vertical and lateral spring constant assigned to the soil spring elements are determined based on the following relation:

$$K_{lateral} = (K_{hd}) \cdot A_{joint} \tag{8}$$

$$K_{vertical} = (K_{vd}) \cdot A_{joint} \tag{9}$$

Where $K_{lateral}$ and $K_{vertical}$ are the soil global lateral and vertical dynamic spring stiffnesses in kip/ft^3 respectively, A_{joint} is the area served by each joint of the base mat elements.

The soil lateral and vertical global dynamic stiffnesses are evaluated based on the elastic half-space theory considering the effect the foundation embedment on the soil stiffness and damping constant. The soil global lateral and vertical stiffnesses are calculated as follows:

$$K_{hd} = 2 \cdot (1+\upsilon) \cdot G_{soil} \cdot \beta_h \cdot \sqrt{B \cdot L} \cdot \eta_h \tag{10}$$

$$K_{vd} = \frac{G_{soil}}{(1-\nu)} \cdot \beta_z \cdot \sqrt{B \cdot L} \cdot \eta_z \quad (11)$$

Where, ν is the soil Poisson's ratio, G_{soil} is the soil shear modulus, β_h is the foundation geometric factors, B, L are the foundation width and length respectively, η_h is the foundation embedment factor for lateral mode of vibration,

$$\eta_h = 1 + 0.55 \cdot (2-\upsilon) \cdot \frac{h_{soil}}{R_{eqv}} \quad (12)$$

$$\eta_z = 1 + 0.6 \cdot (1-\upsilon) \cdot \frac{h_{soil}}{R_{eqv}} \quad (13)$$

η_z is the foundation embedment factor for vertical mode of vibration, h_{soil} is the foundation embedment depth and R_{eqv} is the foundation equivalent radius.

To capture the effect of the equipment weight on the foundation dynamic response, the masses of the crank, cylinder and the motors were added to the model in form of masses lumped at the components support plate. ANSYS element type MASS21 was used to model the component masses. MASS21 is a point element having up to six degrees of freedom: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. A different mass and rotary inertia may be assigned to each coordinate direction. The mass element is defined by a single node; concentrated mass components (Force x Time^2/Length) in the element coordinate directions.

Modal Analysis of the Soil-Foundation System

To determine the foundation vibration characteristics, modal analysis is performed utilizing the finite element model presented above to determine the foundation fundamental natural frequencies and the corresponding modal mass participation factors. The equation of motion for an un-damped system is expressed as follows:

$$[M]\frac{d^2}{dt^2}\{U\} + [K]\{U\} = \{0\} \quad (14)$$

Where [M] is the structure's mass matrix, [K] is the structure's stiffness matrix, {U} is the diaplacement vector where

$$\{U\} = \{\phi_i\} \cdot cos\ (\omega_i t) \quad (15)$$

$\{\phi_i\} = eigenvector\ representing\ the\ mode\ shape\ of\ the\ ith\ frequency$

$\omega_i = ith\ natural\ circular\ frequency\ (rad / sec)$

Tables 2 and 3 show sample of the fundamental natural modes of the soil-foundation system and the corresponding translational and rotational mass participation factors in the global X and Y directions. Shown in these results, part of the dominant natural frequencies of the soil foundation system ranges from 12 Hz (mode No.1) to 15.6 Hz (mode No. 6) with significant mass participation factors excited within this range of frequencies. ACI 351 recommends the foundation natural frequency within +/-20% from the compressor's operating frequency to avoid resonance between the foundation and the machine. The analysis of the soil foundation system shows that modes No. 1 to mode No. 6 fall within the operating range of the compressor with significant mass contribution to these modes.

Table 2: Participation Factor Calculation X Direction

MODE	FREQ (Hz)	PERIOD (Sec)	PARTIC. FACTOR	RATIO	MODE	FREQ (Hz)	PERIOD (Sec)	PARTIC. FACTOR	RATIO
1	12.4446	8.04E-02	4.3558	0.057232	14	34.9272	2.86E-02	3.65E-02	0.00048
2	12.5321	7.98E-02	2.0848	0.027393	15	35.0477	2.85E-02	-0.16054	0.002109
3	14.8916	6.72E-02	27.482	0.361087	16	35.9775	2.78E-02	-0.34718	0.004562
4	15.231	6.57E-02	11.478	0.15081	17	36.8289	2.72E-02	-1.7421	0.022889
5	15.6242	6.40E-02	-76.108	1	18	37.7992	2.65E-02	-0.40175	0.005279
6	15.6641	6.38E-02	-27.81	0.365408	19	38.4719	2.60E-02	-0.63003	0.008278
7	22.3536	4.47E-02	-2.2159	0.029115	20	39.4313	2.54E-02	-1.4689	0.019301
8	24.4395	4.09E-02	-0.64476	0.008472	21	42.0866	2.38E-02	0.27356	0.003594
9	30.7517	3.25E-02	-1.7312	0.022746	22	43.0574	2.32E-02	0.31432	0.00413
10	30.9195	3.23E-02	2.12E-04	0.000003	23	43.2669	2.31E-02	0.22819	0.002998
11	31.9241	3.13E-02	0.4343	0.005706	24	45.0904	2.22E-02	-0.97528	0.012814
12	32.5802	3.07E-02	-0.30677	0.004031	25	48.9339	2.04E-02	-1.03E-02	0.000135
13	33.8849	2.95E-02	-0.33431	0.004393					

Table 3: Participation Factor Calculation Y Direction

MODE	FREQ (Hz)	PARTIC. FACTOR	PARTIC. FACTOR	RATIO	MODE	FREQ (Hz)	PERIOD (Sec)	PARTIC. FACTOR	RATIO
1	12.4446	8.04E-02	0.73631	0.007559	14	34.9272	2.86E-02	-0.10846	0.001113
2	12.5321	7.98E-02	1.4668	0.015059	15	35.0477	2.85E-02	-2.49E-02	0.000256
3	14.8916	6.72E-02	-4.8511	0.049804	16	35.9775	2.78E-02	0.19111	0.001962
4	15.231	6.57E-02	-2.5323	0.025998	17	36.8289	2.72E-02	-0.20989	0.002155
5	15.6242	6.40E-02	34.064	0.349715	18	37.7992	2.65E-02	6.98E-02	0.000716
6	15.6641	6.38E-02	-97.405	1	19	38.4719	2.60E-02	-0.38409	0.003943
7	22.3536	4.47E-02	2.5733	0.026419	20	39.4313	2.54E-02	0.19571	0.002009
8	24.4395	4.09E-02	-0.21252	0.002182	21	42.0866	2.38E-02	-0.48803	0.00501
9	30.7517	3.25E-02	-0.29608	0.00304	22	43.0574	2.32E-02	3.87E-02	0.000397
10	30.9195	3.23E-02	-1.1058	0.011352	23	43.2669	2.31E-02	-0.44105	0.004528
11	31.9241	3.13E-02	-0.51373	0.005274	24	45.0904	2.22E-02	0.12674	0.001301
12	32.5802	3.07E-02	0.18286	0.001877	25	48.9339	2.04E-02	-0.78461	0.008055
13	33.8849	2.95E-02	-0.11518	0.001183					

Figure 4 shows the foundation mode shape at frequencies of 12.446 Hz while Figure 5 shows the foundation mode shape at frequencies 15.6242 Hz.

HARMONIC ANALYSIS OF THE SOIL-FOUNDATION SYSTEM

Due to the presence of unbalance rotating and reciprocating mass and periodic unbalance inertia, dynamic forces and moments are generated in the foundation at the machine bearing supports. The Unbalanced inertia forces result from the acceleration and deceleration of unbalanced reciprocating masses and by the rotation of eccentric masses. Figure 6 shows the kinematics of the compressor pisto and the crank counterweight. The rotating masses consist of the counterweight, the crankpin, the crankpin web, and approximately one-half of the connecting rod.

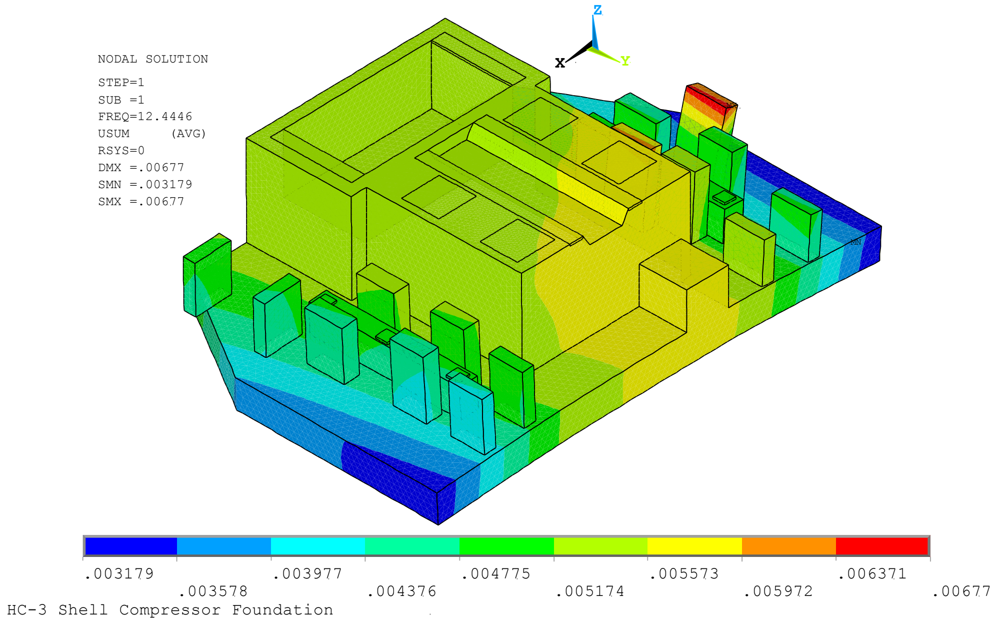

Figure 4. Compressor Foundation Mode Shape at Frequencies 12.446 Hz

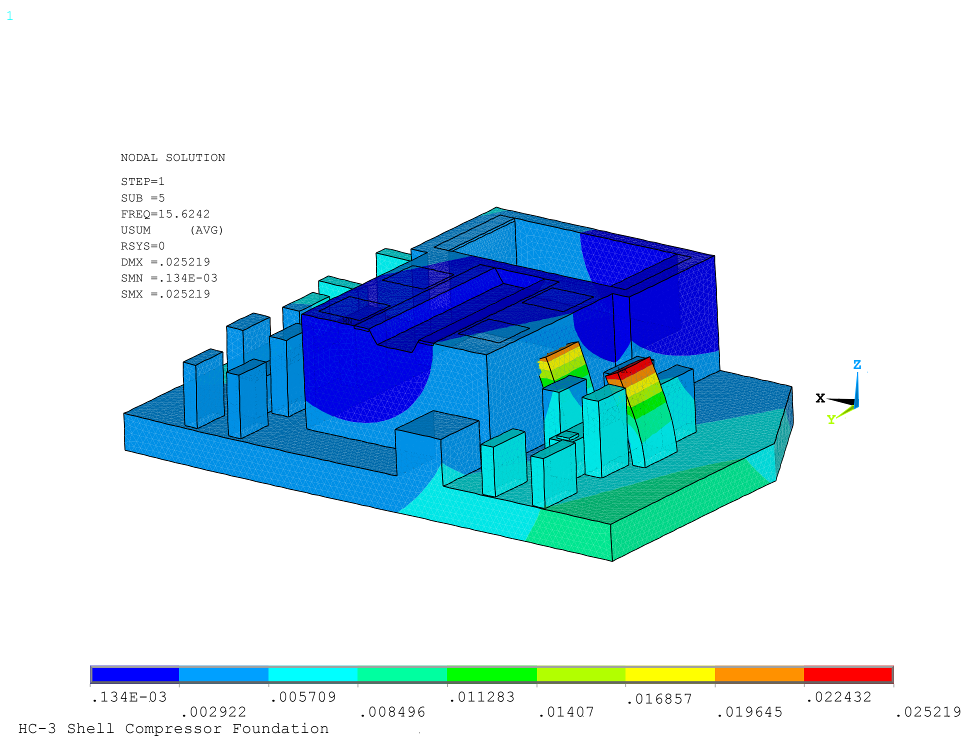

Figure 5. Compressor Foundation Mode Shape at Frequencies 15.6242 Hz

The centrifugal forces created by these masses has the same magnitude for all positions of the crank. The resultant unbalanced rotating force is expressed as follows:

$$F_a = M_a \cdot R \cdot \omega^2 \tag{16}$$

$$M_a = M_r\left(\frac{R_1}{R}\right) + M_c\left(\frac{l_2}{l}\right) \tag{17}$$

where M_r is the mass of the crank rod, M_c is the mass of the connecting rod, R is the length of the cranck rod, R_1 is the distance of its CG (C_{cr}) from center of rotation point O, and ω is the spead of rotation in (rad/sec).

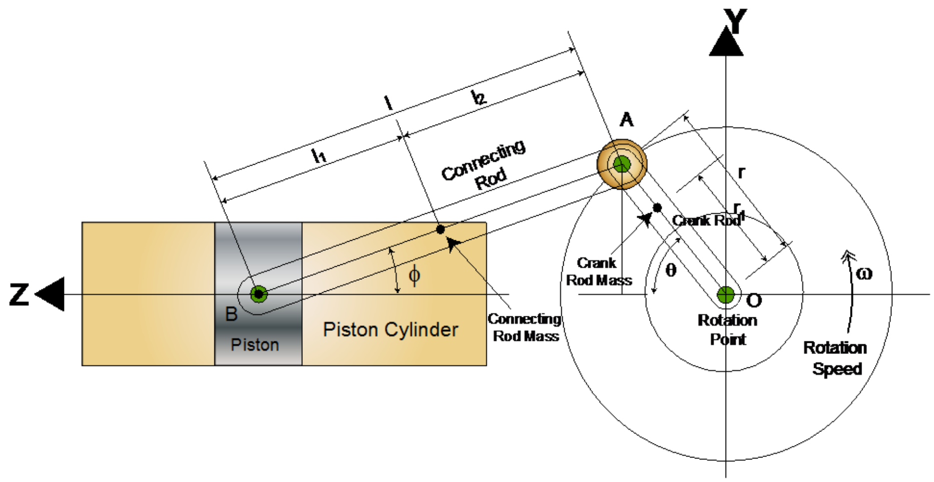

Figure 6. Kinematics of the Compressor Cross-section

The reciprocating force generated along the axis of the cylinder due to the of the reciprocating masses are the piston, piston rod, cross head, and the remaining one half of the connecting rod weight due to the acceleration of the reciprocating masses can be expressed as a Fourier series:

$$F_{piston} = M_p R\,\omega^2\left(cos(\omega t) + \frac{R}{l}\cdot cos(2\omega t)\right) \tag{18}$$

Where M_P is the mass of the piston assembly including piston rod, cross head, etc, R is the length of the crank rod and l is the length of the conneting rod.

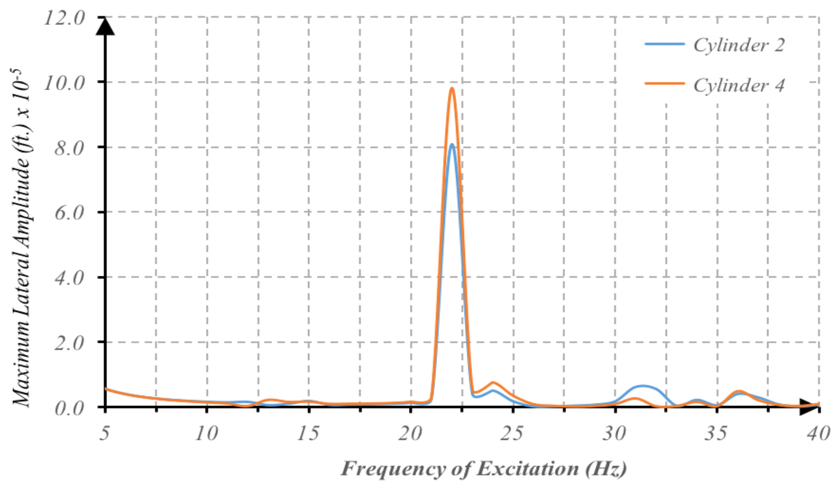

Figure 7. Lateral Harmonic Response of Compressor Cylinder 2 and 4 Supports

Figures 7 & 8 show the sample amplitude response of the foundation at different exciting frequency for the Compressor Cylinder 2 and 4 Supports and Cylinder 4 Discharge Bottle Support Blocks 1 & 3.

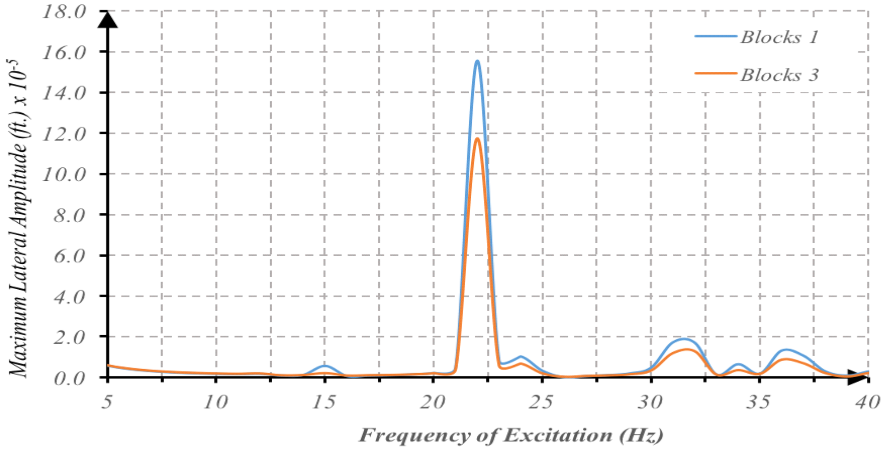

Figure 8. Vertical and Lateral Harmonic Response of Cylinder 4 Discharge Bottle Support Blocks 1 and 3

CONCLUSION

The dynamic assessment of the compressor foundation is determined. A three-dimensional finite element model was developed utilizing the commercial finite element software ANSYS. The effect of the soil-foundation interaction was included in the model, where the soil was modeled as a series of vertical and lateral spring and damper elements. The fundamental natural frequencies and the corresponding mode shapes and mass participation ratios were determined for the soil-foundation system. The response of the soil-foundation under forced excitation of the machine unbalance loading at different exciting frequencies was calculated and presented. Based on the analysis performed, the following is concluded:

1. The response of the foundation system is governed by the response of the individual support piers (blocks) and not the global foundation response, i.e., local modes of vibration. The lateral response of Suction Filter support pier (Block No. 4) is excited at a frequency of 12.4 Hz with almost 5% of the foundation mass being excited (mass participation ratio of 5%). This local mode is close to the compressor operation frequency (13.1Hz).
2. Suction filter support pier No.2 (Block No.5) is laterally excited at 15.2 Hz with a mass participation ratio of 0.5%.
3. At frequency of 15.6 Hz, both the suction filter support piers (Blocks Nos. 4 and 5) are laterally excited with 18% of the foundation mass contributing to this mode.
4. Under harmonic excitation (forced vibration), the foundation global response is resonating with an exciting frequency of approximate 22-23 Hz. However, since the compressor steady state operating frequency is below the foundation resonant frequency (13.1 Hz vs 22-23 Hz) there is no global resonance of the soil-foundation system.
5. The maximum vertical and lateral response of the foundation is 0.014 in. and 0.036 in., respectively. Thus, classifying the foundation to fall within “ Very Good” operational

limit in accordance to Figure 3.10 of ACI 351 (ACI 351, 2004) This classification may lead to some notable vibrations on the foundation

6. The classification of the foundation dynamic operational performance is considered "Very Good" (ACI 351, Figure 3.10 (ACI 351, 2004)). This operational limit has an amplitude limit of 0.156 in, thus inducing notable vibrations. These vibrations will increase the foundation fatigue, thus causing the machine to wear down more quickly than it would have otherwise, and adversely reduce the foundation service-life limit. For machines to run smoothly, the foundation operational limit should be within the range of "Very Smooth" operational limit according to ACI 351 (ACI, 2004). Therefore, to enhance the dynamic performance of the foundation to "Very Smooth operation", it is the recommendation to reduce the lateral response of the individual suction filter support and discharge bottles support. This can be achieved by:
   a. Connect the suction filter support piers and discharge bottles support piers monolithically to the cylinder support piers
   b. Increase the thicknesses of the filter support piers and discharge bottles support to increase their stiffness's and shift their natural frequency from the compressor steady state operation frequency.

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